> *restart*

Cuerda de guitarra

> $Ecua := diff(y(x,t), x\$2) = c^2 \cdot diff(y(x,t), t\$2)$

$$Ecua := \frac{\partial^2}{\partial x^2} y(x,t) = c^2 \left(\frac{\partial^2}{\partial t^2} y(x,t) \right) \tag{1}$$

> $EcuaUno := subs(c^2 = 1, Ecua)$

$$EcuaUno := \frac{\partial^2}{\partial x^2} y(x,t) = \frac{\partial^2}{\partial t^2} y(x,t) \tag{2}$$

> $EcuaDos := eval(subs(y(x,t) = F(x) \cdot G(t), EcuaUno))$

$$EcuaDos := \left(\frac{d^2}{dx^2} F(x) \right) G(t) = F(x) \left(\frac{d^2}{dt^2} G(t) \right) \tag{3}$$

> $EcuaSep := \frac{lhs(EcuaDos)}{F(x) \cdot G(t)} = \frac{rhs(EcuaDos)}{F(x) \cdot G(t)}$

$$EcuaSep := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \frac{\frac{d^2}{dt^2} G(t)}{G(t)} \tag{4}$$

> $EcuaXalpha := lhs(EcuaSep) = \text{alpha}$

$$EcuaXalpha := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha \tag{5}$$

> $EcuaTalpha := rhs(EcuaSep) = \text{alpha}$

$$EcuaTalpha := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \tag{6}$$

> $CondFrontera := y(0,t) = 0, y(L,t) = 0$

$$CondFrontera := y(0,t) = 0, y(L,t) = 0 \tag{7}$$

> $CondInicial := y(0,0) = \left(\frac{a}{\frac{L}{2}} \right) \cdot x, y(1,0) = 2 \cdot a - \left(\frac{a}{\frac{L}{2}} \right) \cdot x : CondInicial[1];$
> $CondInicial[2];$

$$y(0,0) = \frac{2\,a\,x}{L}$$

$$y(1,0) = 2\,a - \frac{2\,a\,x}{L} \tag{8}$$

> $DerCondIniaicl := \mathrm{D}[t](y(x,0)) = 0$

$$DerCondIniaicl := \mathrm{D}_t(y(x,0)) = 0 \tag{9}$$

> $L := 1$

$$L := 1 \tag{10}$$

> $CondInicial$

$$y(0,0) = 2\,a\,x, y(1,0) = -2\,a\,x + 2\,a \tag{11}$$

Para alpha=cero

> $EcuaXcero := subs(\text{alpha}=0, EcuaXalpha)$

$$EcuaXcero := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 0 \tag{12}$$

> $EcuaTcero := subs(\text{alpha}=0, EcuaTalpha)$

$$EcuaTcero := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = 0 \tag{13}$$

> $SolXcero := dsolve(EcuaXcero)$

$$SolXcero := F(x) = c_1 x + c_2 \tag{14}$$

> $ComprobarUno := subs(x=0, rhs(SolXcero) = 0)$

$$ComprobarUno := c_2 = 0 \tag{15}$$

> $ComprobarDos := subs(x=1, c_2=0, rhs(SolXcero) = 0)$

$$ComprobarDos := c_1 = 0 \tag{16}$$

> $F(x) = 0$

$$F(x) = 0 \tag{17}$$

> $y(x, t) = 0$

$$y(x, t) = 0 \tag{18}$$

Para alpha positiva

> $EcuaXpos := subs(\text{alpha}=\beta^2, EcuaXalpha)$

$$EcuaXpos := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \beta^2 \tag{19}$$

> $EcuaTpos := subs(\text{alpha}=\beta^2, EcuaTalpha)$

$$EcuaTpos := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \beta^2 \tag{20}$$

> $SolXpos := dsolve(EcuaXpos)$

$$SolXpos := F(x) = c_1 e^{\beta x} + c_2 e^{-\beta x} \tag{21}$$

> $SistemaPos := eval(subs(x=0, rhs(SolXpos) = 0)), eval(subs(x=1, rhs(SolXpos) = 0)) :$
> $SistemaPos[1]; SistemaPos[2];$

$$c_1 + c_2 = 0$$

$$c_1 e^{\beta} + c_2 e^{-\beta} = 0 \tag{22}$$

> $ParaPos := solve(\{SistemaPos\}, \{c_1, c_2\})$

$$ParaPos := \{c_1 = 0, c_2 = 0\} \tag{23}$$

> $F(x) = 0$

(24)

$$F(x) = 0 \quad \textbf{(24)}$$

> $y(x, t) = 0$

$$y(x, t) = 0 \quad \textbf{(25)}$$

para alpha negativa

>

> $EcuaXneg := eval\left(subs\left(\text{alpha} = -\beta^2, EcuaXalpha\right)\right)$

$$EcuaXneg := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -\beta^2 \quad \textbf{(26)}$$

> $EcuaTneg := eval\left(subs\left(\text{alpha} = -\beta^2, EcuaTalpha\right)\right)$

$$EcuaTneg := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = -\beta^2 \quad \textbf{(27)}$$

> $SolXneg := dsolve(EcuaXneg)$

$$SolXneg := F(x) = c_1 \sin(\beta x) + c_2 \cos(\beta x) \quad \textbf{(28)}$$

> $ComprobarCinco := simplify(subs(x = 0, rhs(\ SolXneg) = 0))$

$$ComprobarCinco := c_2 = 0 \quad \textbf{(29)}$$

> $ComprobarSeis := subs\left(c_2 = 0, \text{beta} = n \cdot \text{Pi}, rhs(SolXneg) = 0\right)$

$$ComprobarSeis := c_1 \sin(n \pi x) = 0 \quad \textbf{(30)}$$

> $c_1 \neq 0$

$$c_1 \neq 0 \quad \textbf{(31)}$$

> $EcuaXfinal := subs\left(\text{alpha} = -(n \cdot \text{Pi})^2, EcuaXalpha\right)$

$$EcuaXfinal := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -n^2 \pi^2 \quad \textbf{(32)}$$

> $SolXfinal := subs\left(c_1 = 1, c_2 = 0, dsolve(EcuaXfinal)\right)$

$$SolXfinal := F(x) = \sin(n \pi x) \quad \textbf{(33)}$$

> $EcuaTfinal := subs\left(\text{alpha} = -(n \cdot \text{Pi})^2, EcuaTalpha\right)$

$$EcuaTfinal := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = -n^2 \pi^2 \quad \textbf{(34)}$$

> $SolTfinal := dsolve(EcuaTfinal)$

$$SolTfinal := G(t) = c_1 \sin(n \pi t) + c_2 \cos(n \pi t) \quad \textbf{(35)}$$

> $SolGralfinal := y(x, t) = rhs(SolXfinal) \cdot rhs(SolTfinal)$

$$SolGralfinal := y(x, t) = \sin(n \pi x) \left(c_1 \sin(n \pi t) + c_2 \cos(n \pi t)\right) \quad \textbf{(36)}$$

> $SolGralFourier := y(x, t) = Sum(\sin(n \cdot \text{Pi} \cdot x) \cdot (b[n] \cdot \cos(n \cdot \text{Pi} \cdot t) + a[n] \cdot \sin(n \cdot \text{Pi} \cdot t)), n = 1$
> $..\text{infinity})$

$$SolGralFourier := y(x,t) = \sum_{n=1}^{\infty} \sin(n\pi x)\left(b_n \cos(n\pi t) + a_n \sin(n\pi t)\right) \quad \textbf{(37)}$$

> $SolGralTotal := eval(subs(t=0, SolGralFourier))$

$$SolGralTotal := y(x,0) = \sum_{n=1}^{\infty} \sin(n\pi x)\, b_n \quad \textbf{(38)}$$

> $b[n] := subs\left(\sin(n\cdot \mathrm{pi}) = 0, \cos(n\cdot \mathrm{Pi}) = (-1)^n, simplify\left(\left(\frac{1}{\frac{1}{2}}\right)\cdot int\left(\left(\frac{\left(\frac{1}{100}\right)}{\frac{1}{2}}\cdot x\cdot \sin(n\cdot \mathrm{Pi}\cdot x)\right), x = 0..\frac{5}{10}\right) + \left(\frac{1}{\frac{1}{2}}\right)\cdot int\left(\left(\left(\frac{2}{100}\right) - \frac{\left(\frac{1}{100}\right)}{\left(\frac{1}{2}\right)}\cdot x\right)\cdot \sin(n\cdot \mathrm{Pi}\cdot x), x = \frac{5}{10}..1\right)\right)\right)$

$$b_n := \frac{2\sin\left(\frac{n\pi}{2}\right) - \sin(n\pi)}{25\, n^2 \pi^2} \quad \textbf{(39)}$$

> $isolate(eval(rhs(subs(t=0, diff(SolGralFourier, t))) = 0), a[n])$

$$\sum_{n=1}^{\infty} \sin(n\pi x)\, a_n\, n\pi = 0 \quad \textbf{(40)}$$

> $a[n] := 0$

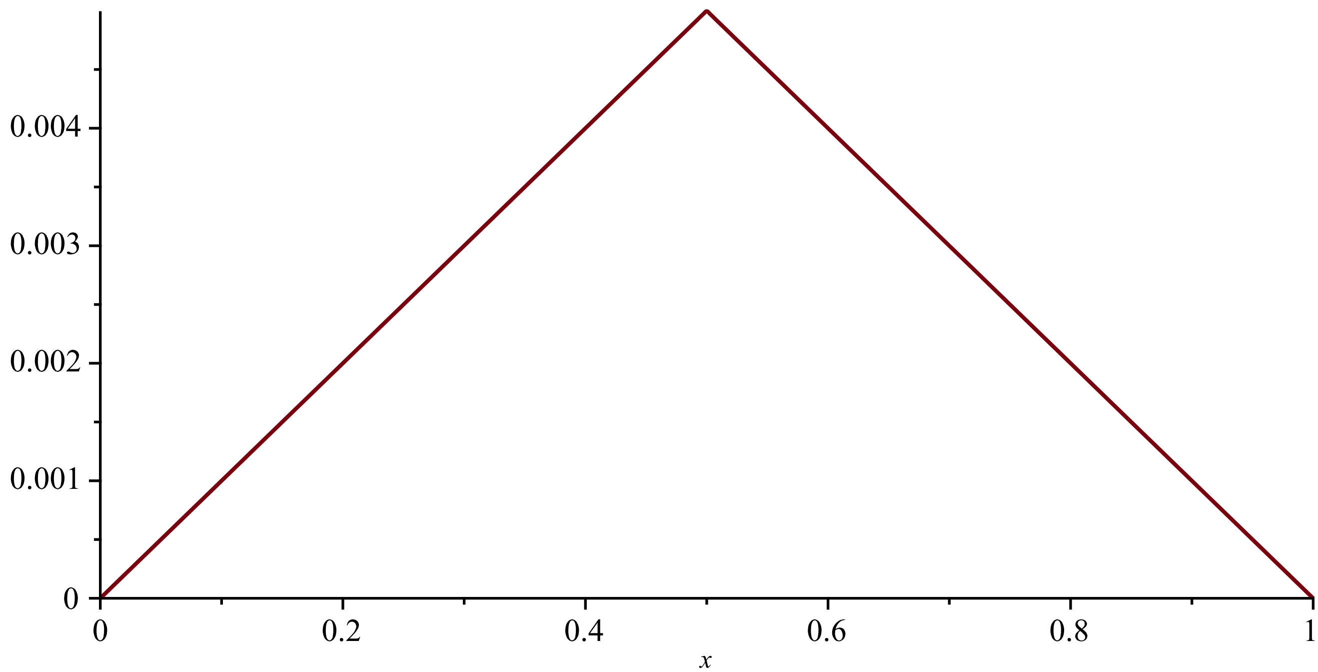
$$a_n := 0 \quad \textbf{(41)}$$

> $SolGralFourier$

$$y(x,t) = \sum_{n=1}^{\infty} \frac{\sin(n\pi x)\left(2\sin\left(\frac{n\pi}{2}\right) - \sin(n\pi)\right)\cos(n\pi t)}{25\, n^2 \pi^2} \quad \textbf{(42)}$$

> $SolPart500 := y(x,t) = sum\left(\frac{\sin(n\cdot \mathrm{Pi}\cdot x)\cdot \sin\left(\frac{n\cdot \mathrm{Pi}}{2}\right)\cdot \cos(n\cdot \mathrm{Pi}\cdot t)}{25\cdot n^2\cdot \mathrm{Pi}^2}, n = 1..500\right):$

> $plot(subs(t=0, rhs(SolPart500)), x = 0..1)$



> $with(plots):$

> $animate(rhs(SolPart500), x=0..1, t=0..4, frames=150, view=[0..1, -0.01..0.01])$